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A mixed phase of SUSY gauge theories from a-maximization

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Abstract: We study $\mathcal{N} = 1$ supersymmetric $\mathrm{SU}(N)$ gauge theories with an antisymmetric tensor and F flavors using the recent proposal of a-maximization by Intriligator and Wecht. This theory had previously been studied using the method of "deconfinement," but such an analysis was not conclusive since anomalous dimensions in the non-perturbative regime could not be calculated. Using a-maximization we show that for a large range of F the theory is at an interacting superconformal fixed point. However, we also find evidence that for a range of F the theory in the IR splits into a free "magnetic" gauge sector and an interacting superconformal sector.

Keywords: Renormalization Group, Supersymmetry and Duality, Supersymmetric Effective Theories.

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1. Introduction

Finding the low-energy effective lagrangian for a gauge theory in the strong coupling regime is a virtually impossible task unless one appeals to lattice computations. However, if one considers theories with a large symmetry group then the symmetries could possibly be powerful enough to restrict the structure of the low-energy effective lagrangian. This is what happens in supersymmetric (SUSY) theories. Unbroken supersymmetry implies that part (or all) of the lagrangian is governed by holomorphic objects, which can sometimes be uniquely fixed based on symmetry arguments and weak coupling limits. For $\mathcal{N} = 2$ theories the entire theory is determined by a holomorphic quantity, the prepotential, which allows one to solve the theory exactly [1] in the IR. In $\mathcal{N} = 1$ theories the entire lagrangian is not governed by a holomorphic object, but only the superpotential. The fact that the superpotential is holomorphic in $\mathcal{N} = 1$ theories still allows one to make powerful statements about non-perturbative physics and often lets one find the vacuum structure and phase of the theory [2]. The possible phases for theories with a small enough matter content are found to be confining (with or without chiral symmetry breaking), a pure abelian Coulomb phase (analogous to the $\mathcal{N} = 2$ theories), or the gauge group could be broken via a dynamically generated superpotential. Theories with these phases have been completely classified in [3, 4, 5]. Once the matter content of a SUSY gauge theory is large enough, the known possible phases are an interacting superconformal fixed point (non-abelian Coulomb phase) or free "electric" or "magnetic" phases. The major tool used to study these phases is Seiberg duality [6], which is based on holomorphy and 't Hooft anomaly matching. Seiberg duality is the IR equivalence of two different SUSY gauge theories with the same flavor symmetries and holomorphic invariants. This is often a strong-weak duality, which means in the regime where one of the theories is strongly coupled the dual is weakly coupled. The canonical examples of Seiberg duality typically contain three regimes: where the electric theory is IR free, while the magnetic theory is strongly coupled (free electric theory), where both electric and magnetic theories are interacting, but they correspond to the same IR fixed point (non-abelian Coulomb phase), and where the dual is IR free while the electric theory is strongly coupled (free magnetic phase). However, there are relatively few examples of Seiberg duality where the IR behavior is known, and thus one can only guess the right low-energy description of most of the $\mathcal{N} = 1$ theories.

An important step in finding a general prescription to determine the low-energy description of most $\mathcal{N} = 1$ gauge theories has recently been made by Intriligator and Wecht [7]. The key ingredient is that when a SUSY theory is at a fixed point, it necessarily has a larger space-time symmetry group, the superconformal group. A particular $\mathrm{U}(1)_R$ symmetry plays a special role since its corresponding R-charge is one of the generators of the superconformal group. From the superconformal algebra it follows [8, 9] that the R-charge and the dimension Δ of a chiral operator $\mathcal{O}$ satisfy

$$\frac{3}{2}R(\mathcal{O}) = \Delta(\mathcal{O})\,. \tag{1.1}$$

Therefore we can determine the anomalous dimensions of the entire chiral ring from the fact that $\Delta \equiv 1+\gamma/2$ if we can determine their R-charges. Thus we can "solve" the gauge theory if we can determine the R-symmetry $\mathrm{U}(1)_R \subset \mathrm{SU}(2,2|1)$ of the superconformal algebra. This is an easy problem in SUSY QCD where there is no ambiguity in determining the $\mathrm{U}(1)_R$ symmetry. However when there are additional fields there are additional U(1) flavor symmetries and and one can form a linear combination of these anomaly free U(1)'s. It is then not clear what principle will determine which of these linear combinations will be the preferred $\mathrm{U}(1)_R$ that appears in the superconformal algebra. This is the problem that has been recently solved by Intriligator and Wecht [7] through a process called a-maximization. They found that the R-symmetry appearing in the superconformal algebra is the one that maximizes a central charge called a. A brief review of this process will be presented at the beginning of the next section.

The ability to find the superconformal $\mathrm{U}(1)_R$ symmetry is a major step forward in exact results in SUSY gauge theories. However, finding the superconformal $\mathrm{U}(1)_R$ still does not solve the entire theory since there is no way from a-maximization to determine where the superconformal phase ends. The process of a-maximization has only been implemented in a few cases such as for $\mathrm{SU}(N)$ gauge theories with one and two adjoints and various superpotential perturbations [7, 10, 11], all of which are vector-like theories (it has also been examined in the case of general theories in a different framework [12]). However, some of the most interesting SUSY gauge theories are the chiral theories, since these are the ones that can lead to dynamical supersymmetry breaking. The simplest models of dynamical SUSY breaking usually utilize a gauge theory with an antisymmetric tensor and some number of flavors [13, 14, 15]. Therefore, a lot of effort was expended during the nineties to try to understand the dynamics of such theories [16]–[21]. For a small number of flavors the dynamics of the theory is well-understood [17, 19, 4], however one could not conclusively find the low-energy phase of such a theory for an arbitrary number of flavors.

In light of the new developments we return in this paper to the study of the dynamics of a supersymmetric SU(N) gauge theory with one two-index antisymmetric tensor, F fundamentals, and $N+F-4$ antifundamentals and no tree-level superpotential. The new methods will allow us to finally pin down the phase structure of this model. In studying this theory we implement a-maximization first directly, and then consider a dual of this theory based on the method of "deconfinement" [18]. The process of "deconfinement" allows one to come up with another strongly coupled description of the original theory using only fundamental fields which can then be dualized using ordinary Seiberg duality. Studying this deconfined dual will let us explore what happens to the theory when some of the gauge invariants in the electric theory go free. The dual of this theory using "deconfinement" has been previously studied in [19, 21]. However, since the superconformal R-symmetry could not be found before the method of a-maximization was known, one could not draw definite conclusions about the phase of the theory even using the deconfined dual.

One of the advantages of considering the deconfined dual in the case of the antisymmetric tensor is that the fields that go free as the number of flavors is reduced will be elementary fields in the dual theory. Therefore the procedure suggested in [10] of eliminating the contributions of the free field from a in the a-maximization procedure is straightforward to carry out. One can also check explicitly if a field going free would also imply the existence of a new phase or not. The deconfined dual has a product group structure. In [21] hints were found that for $F=5$ and for $N>6$ a new type of mixed phase occurs where one of the gauge groups remains at a superconformal fixed point whereas the other group becomes IR free. In this paper we use a-maximization in combination with "deconfinement" and find strong evidence for the fact that this new mixed phase exists and determine exactly when it occurs. What this implies is that the original electric theory after a certain point ceases to be a good description of the physics and actually splits into two sectors: an interacting non-abelian Coulomb phase and a co-existing free magnetic phase. The structure of the paper is as follows: in section 2 we outline the original theory and give the results of a-maximization for this "electric" theory. In section 3 we explain the method of "deconfinement" and show how to use it to find a dual description of the electric theory. In section 4 we then implement a-maximization in the deconfined dual theory, show explicitly how the decoupling of the free fields happens in the dual description, and discuss the arguments for the appearance of a mixed phase in this theory.

2. a-maximization in the electric theory

Before we start analyzing the SU(N) theory with an antisymmetric tensor, we will briefly review the central charge a and a-maximization for those not familiar with the original paper of Intriligator and Wecht [7]. In a SUSY gauge theory the trace anomaly of the stress-energy tensor, $T^{\mu\nu}$, has both internal contributions (from the gauge sector) and external contributions from external background sources that are coupled to currents in the theory. The central charge a of a four-dimensional superconformal gauge theory is the coefficient of the contribution from an external supergravity background. The definition of

a comes from coupling the stress energy tensor to a background metric $g_{\mu\nu}(x)$ which then shows up in the trace anomaly as

$$T^\mu_\mu = \Theta \sim \frac{1}{g^3}\tilde{\beta}(F^a_{\mu\nu})^2 - a(g)(R_{\mu\nu\rho\sigma})^2 + \cdots \tag{2.1}$$

where g is the gauge coupling, $\tilde{\beta}$ is the numerator of the exact NSVZ β function [22], $F^a_{\mu\nu}$ is the gauge field strength, and R is the curvature tensor whose square is the Euler density. The central charge a was conjectured by Cardy [23] to satisfy a four dimensional version of the Zamolodchikov c-theorem [24]: $a_{IR} < a_{UV}$. The connection between the $\mathrm{U}(1)_R$ symmetry that is in the superconformal algebra and a is that a can be expressed in terms of 't Hooft anomalies of this particular R-symmetry [25, 26]. The relation between a and the 't Hooft anomalies is

$$a = \frac{3}{32}\left(3\,\mathrm{Tr}\,R^3 - \mathrm{Tr}\,R\right). \tag{2.2}$$

This relation still does not tell us what the superconformal $\mathrm{U}(1)_R$ symmetry is since neither side of eq. (2.2) is fixed at this point.

Let us consider a trial R-symmetry made up of some arbitrarily chosen initial R-symmetry R_0 and the various additional $\mathrm{U}(1)$ symmetries Q_I of the global symmetry group of the theory

$$R_{\mathrm{trial}} = R_0 + \sum_i s_I Q_I \tag{2.3}$$

where s_I are arbitrary real coefficients that tell us the admixture of symmetries making up our trial R-symmetry. What Intriligator and Wecht have shown is that the s_I corresponding to the linear combination that gives the superconformal R-symmetry, $\hat{s}_I$, come from maximizing the central charge a_{trial}. a_{trial} is constructed by using the trial R-symmetry in eq. (2.3) with eq. (2.2). The condition that a is maximized implies that the first derivatives of a_{trial} with respect to the s_I vanish which implies

$$\frac{\partial a_{\mathrm{trial}}}{\partial s_I} = \frac{3}{32}\left(9\,\mathrm{Tr}\,R^2_{\mathrm{trial}} Q_I - \mathrm{Tr}\,Q_I\right) = 0\,. \tag{2.4}$$

Thus the first condition of a-maximization is

$$9\,\mathrm{Tr}(R^2 Q_I) = \mathrm{Tr}\,Q_I\,. \tag{2.5}$$

To find a local maximum the second condition is that the matrix of second derivatives

$$\frac{\partial^2 a_{\mathrm{trial}}}{\partial s_I \partial s_J} = \frac{27}{16}\,\mathrm{Tr}\,R_{\mathrm{trial}} Q_I Q_J < 0 \tag{2.6}$$

is negative-definite. Intriligator and Wecht showed in [7] that eqs. (2.5) and (2.6) were always true for any unitary superconformal field theory. Therefore maximizing a over the space of possible R-symmetries determines the superconformal $\mathrm{U}(1)_R$.

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The process of a-maximization relies on being able to identify the superconformal R-symmetry from the weakly coupled UV fixed point, assuming that the global symmetries of the IR superconformal theory match those of the UV theory. However, in many cases there are accidental symmetries in the IR, which can also be part of the superconformal R-symmetry. For instance such an accidental symmetry appears when one of the gauge invariants becomes a free field. At such points a-maximization could conceivably break down, since the theory will not necessarily remain in the superconformal phase. For example a field going free could signal the appearance of a free magnetic phase. The only way to unambiguously decide what is exactly happening at such points is if one has a weakly coupled dual at hand, which is usually not the case.

We are interested in the low-energy behavior of a supersymmetric $\mathrm{SU}(N)$ gauge theory with one two-index antisymmetric tensor, F fundamentals and $F+N-4$ antifundamentals. We would like to use the method of a-maximization which is applicable in the superconformal phase. Thus we first look for the Banks-Zaks fixed point [27] which occurs for $F = 2N - 3 - \epsilon N$. To simplify the expressions we will work explicitly in the large-N, F limit with $x \equiv N/F$ held fixed. The theory has the following transformation properties under the non-abelian flavor symmetries and the superconformal R-symmetry:

	$\mathrm{SU}(N)$	$\mathrm{SU}(F)$	$\mathrm{SU}(F+N-4)$	$\mathrm{U}(1)_R$
Q	$\square$	$\square$	$\mathbf{1}$	$R(Q)$
$\overline{Q}$	$\overline{\square}$	$\mathbf{1}$	$\square$	$R(\overline{Q})$
A	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{1}$	$\mathbf{1}$	$R(\mathrm{A})$

(2.7)

The vanishing of the NSVZ β function is necessary for the theory to be at a superconformal fixed point. This condition is equivalent to the cancellation of the $\mathrm{SU}(N)^2\,\mathrm{U}(1)_R$ anomaly for the superconformal R-symmetry, and implies that

$$R(\mathrm{A}) = \frac{2 - R(Q) - (x+1)R(\overline{Q})}{x}\,. \tag{2.8}$$

Applying the a-maximization procedure we find that

$$R(Q) = R(\overline{Q}) = -\frac{12 - 9x^2 + \sqrt{x^2(-4 + x(73x - 4))}}{3(-4 + (x-4)x)}\,. \tag{2.9}$$

The flow of the R-charges as a function of x is shown in figure 1 which starts at a value of $x = .5$, corresponding to the Banks-Zaks fixed point in the large-N, F limit. One should note that the R-charges of Q and $\overline{Q}$ are the same even though the theory is chiral. This corresponds to the fact that to all orders in perturbation theory the anomalous dimensions for Q and $\overline{Q}$ have to agree in the absence of a superpotential, since gauge interactions do not distinguish between the two fields. The R-charges in figure 1 will be modified for $x > x_M \sim 2.95367$ due to unitarity constraints that we will discuss.

The chiral ring of this theory is made up of two types of mesons, $M = Q\overline{Q}$ and $H = \overline{Q}A\overline{Q}$, as well as baryons of the form $B_k = Q^k A^{\frac{N-k}{2}}$ (for k, N both even or odd and $k \leq min(N,F)$) and $B = \overline{Q}^N$. Unitarity constrains the dimensions of the operators

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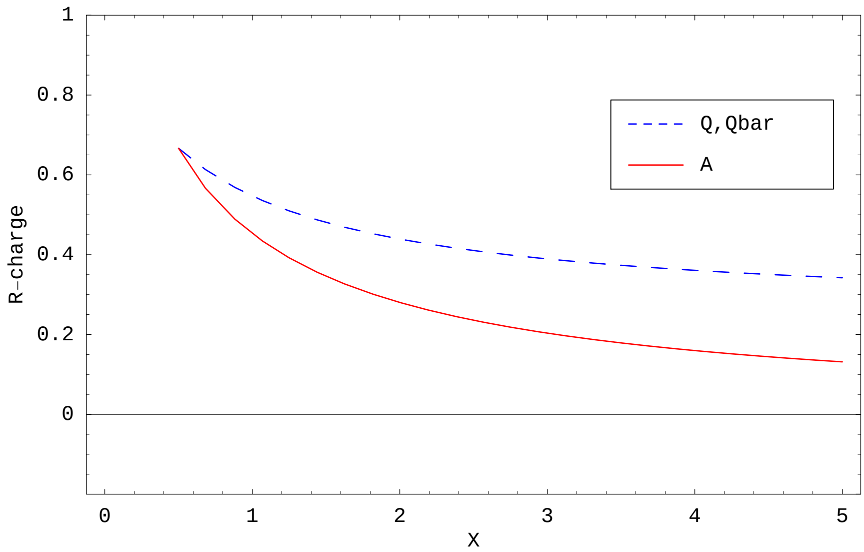

Figure 1: The naive R-charges of the fields are plotted as a function of x, the dashed line represents the R-charges of Q, $\overline{Q}$ and the solid line is the anti-symmetric tensor A. For $x > x_M \sim 2.95367$ the R-charges will be modified when taking into account unitarity constraints.

in the chiral ring to be greater than one. A possible signal for a theory to leave the superconformal phase is when there is an apparent violation of the unitarity constraint (for example in SUSY QCD, the meson becoming a free field signals the onset of the free magnetic phase). In the theory with the antisymmetric tensor under consideration here the smallest invariant (in terms of number of fields) is M, therefore this field is likely to go free first as x is increased which is what we find. We find the point at which the meson becomes a free field is at

$$x = x_M = \frac{4}{9}\left(4+\sqrt{7}\right) \sim 2.95367\,. \tag{2.10}$$

There are then two possibilities: either the theory is out of the superconformal phase and a-maximization should no longer be used, or it is also possible that at the point where M first appears to violate the unitarity bound the meson becomes a free field while the other members of the chiral ring are still interacting. It is impossible to decide just based on the electric theory which of these possibilities actually occurs, but for now we will assume that it is the latter case (that is M becomes free while the other fields remain interacting). We will see more compelling evidence for this from the deconfined dual description in the the next section.

If the meson becomes a free field, there will be an additional U(1) symmetry not present in the UV description which can mix into the superconformal R-symmetry and which will ensure that the R-charge of M is $2/3$. One then subtracts the contribution of the meson from the a used originally and re-maximizes to find the flow for values of x greater than those in eq. (2.10) as first described in [10]. Following the procedure set out in [10] once the meson M goes free we construct a new $a_{\rm int}$ made up of only the assumed interacting

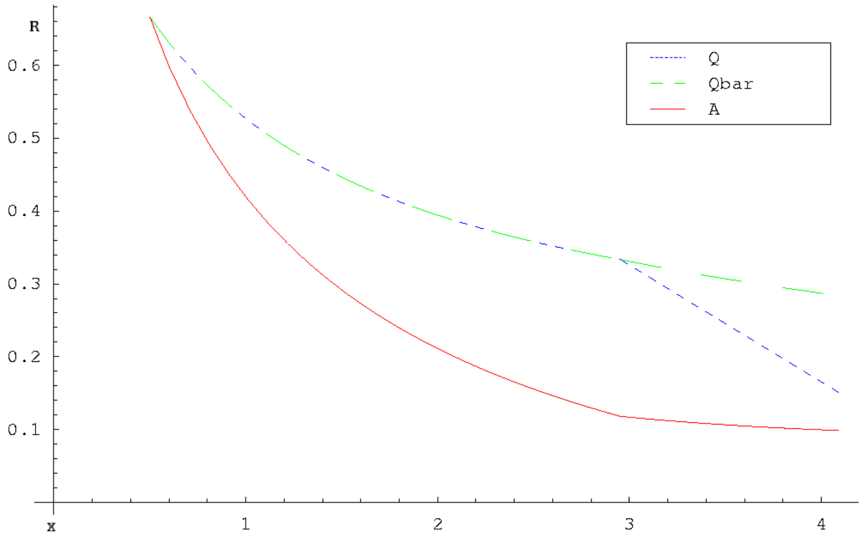

Figure 2: The R-charges of the fields are plotted as a function of x taking into account that M went free. The short dashed line represents the R-charges of Q, and the long dashed line represents $\overline{Q}$ while the solid line is the anti-symmetric tensor A.

sector of the theory

$$a_{\rm int} = a_0 - a(R(M)) \tag{2.11}$$

$$a_{\rm int} = a_0 - \frac{3}{32}F(F+N-4)\big(3(R(Q)+R(\overline{Q})-1)^3 - (R(Q)+R(\overline{Q})-1)\big)\,, \tag{2.12}$$

where a_0 is the original a of the theory. Maximizing $a_{\rm int}$ now determines the R-charges of Q, $\overline{Q}$ and A for $x > x_M$. Because the analytical expressions for the R-charges become too complicated to present here we simply plot the results in figure 2.

The second meson $H = \overline{Q}A\overline{Q}$ also goes free when $x = x_H \sim 4.08952$. Therefore in figure 2 for $x > x_H$ the R-charges will be modified when taking into account unitarity constraints. That H goes free at this value of x depends upon the large-N limit, for $N < 8$ H will not become a free field before the theory confines for $F < 5$. When H goes free one might expect that the same thing happens as when M went free, i.e. it decouples and the rest of the theory remains interacting. We do not continue this process in this “electric” description because in a dual “deconfined” description we believe H going free signals a change from a purely interacting non-abelian Coulomb phase into a mixed phase that we will describe in section 4.

3. Deconfinement

No weakly coupled dual with one gauge group is known for the theory under consideration. This is unfortunate since as was pointed out in [10] one does not know when a-maximization breaks down and a free magnetic phase occurs without a Seiberg dual. There is the possibility though that there is another strongly coupled description of the same physics. We can find such a strongly coupled description of the SU(N) gauge theory in question

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if we view the antisymmetric tensor as a composite coming from an s-confining (confining without chiral symmetry breaking and with a confining superpotential) Sp gauge group: this idea is known as "deconfinement" [18]. It is a rather straightforward process since s-confining SUSY gauge theories are well documented and classified [4]. Once one has a new strongly coupled description of the physics in terms of only fundamental representations of SU(N) one applies the usual Seiberg dualities to find further dual descriptions of the physics. As we will see after applying Seiberg duality, instead of having two groups where one is in a non-abelian Coulomb phase and the other is confining, both will be in a non-abelian coulomb phase for small enough x. With that in mind let us now look at the details of this procedure for our theory in question which has been examined in [19, 21]. One can skip ahead to the final (second) dual description since that is all we will use here but we have included the intermediate steps for completeness.

We start with the SU(N) gauge theory with an antisymmetric tensor [19, 21] for odd N and with $F \geq 5$ flavors:

	SU(N)	SU(F)	SU($F+N-4$)
Q	$\square$	$\square$	$\mathbf{1}$
$\overline{Q}$	$\overline{\square}$	$\mathbf{1}$	$\square$
A	$\overset{\square}{\square}$	$\mathbf{1}$	$\mathbf{1}$

(3.1)

Deconfinement means that instead of considering the above theory with the antisymmetric tensor, one imagines that this antisymmetric tensor is a composite meson of another strongly interacting gauge group that confined before the SU(N) group became strongly interacting. Thus we assume that there is a gauge group G which has a weakly gauged flavor symmetry SU(N). Since we want the meson to be in an antisymmetric representation of this SU(N), the gauge group should be chosen to be an Sp group. It is well-known that Sp(M) is s-confining (that is confining without chiral symmetry breaking) if the number of fundamentals under Sp(M) are $F = M + 4$. However, this confining group will generate a superpotential for the confined meson. In order to eliminate this superpotential *after* confinement one needs to add an additional superpotential term to the theory *before* confinement. The role of this term in the superpotential will be to set some fields to zero after confinement and thereby eliminating the entire confining superpotential. The details of this procedure for deconfining arbitrary two-index representation are described in [20]. Here we just repeat the main steps leading to the final dual description that we will be using.

The deconfined dual description of this theory is then found by taking A to be a composite meson of an s-confining Sp($N-3$) theory

	SU(N)	Sp($N-3$)	SU(F)	SU($N+F-4$)
Y	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$
Z	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\mathbf{1}$
$\overline{P}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Q	$\square$	$\mathbf{1}$	$\square$	$\mathbf{1}$
$\overline{Q}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\square$

(3.2)

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with a superpotential

$$W = YZ\overline{P}\,. \tag{3.3}$$

When the $\mathrm{Sp}(N-3)$ group confines, the superpotential becomes a mass term for one of the meson components, which eliminates the entire confining superpotential.

The $\mathrm{SU}(N)$ group has $N+F-3$ flavors so we can use Seiberg duality for SU groups to find a dual for this deconfined theory:

	$\mathrm{SU}(F-3)$	$\mathrm{Sp}(N-3)$	$\mathrm{SU}(F)$	$\mathrm{SU}(N+F-4)$
y	$\square$	$\square$	$\mathbf{1}$	$\mathbf{1}$
$\overline{p}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
q	$\square$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$
$\overline{q}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\overline{\square}$
M	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$
L	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\square$
B_1	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$

(3.4)

with

$$W = Mq\overline{q} + B_1 q\overline{p} + Ly\overline{q}\,. \tag{3.5}$$

In this first dual we have an $\mathrm{Sp}(N-3)$ group with $N+2F-7$ fundamentals. We can then use Seiberg duality for Sp groups [28] to find an $\mathrm{Sp}(2F-8)$ dual, which after integrating out massive fields has the following content

	$\mathrm{SU}(F-3)$	$\mathrm{Sp}(2F-8)$	$\mathrm{SU}(F)$	$\mathrm{SU}(N+F-4)$
$\tilde{y}$	$\overline{\square}$	$\square$	$\mathbf{1}$	$\mathbf{1}$
$\overline{p}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
q	$\square$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$
M	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$
l	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\overline{\square}$
B_1	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$
a	$\substack{\square\\\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
H	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\substack{\square\\\square}$

(3.6)

and superpotential

$$W = Mql\tilde{y} + Hll + B_1 q\overline{p} + a\tilde{y}\tilde{y}\,. \tag{3.7}$$

In the final dual description we see that both $M = Q\overline{Q}$ and $H = \overline{Q}A\overline{Q}$ are mapped to fundamental fields. This is important, because in the process of a-maximization it is easier to decide what happens to the theory when fundamental fields rather than composite objects become free. To even have the possibility of a weakly coupled description, a field that becomes free must be fundamental in that description.

One can similarly find a dual for the case when N is even by following the same steps outlined above. The final dual will be:

	$\mathrm{SU}(F-2)$	$\mathrm{Sp}(2F-8)$	$\mathrm{SU}(F)$	$\mathrm{SU}(N+F-4)$	$\mathrm{SU}(2)$
$\tilde{y}$	$\overline{\square}$	$\square$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\overline{p}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\square$
q	$\square$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$	$\mathbf{1}$
M	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\square$	$\mathbf{1}$
l	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\overline{\square}$	$\mathbf{1}$
S	$\mathbf{1}$	$\mathbf{1}$	$\square$	$\mathbf{1}$	$\square$
a	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
H	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$\mathbf{1}$
B_0	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

(3.8)

with the superpotential

$$W = Mql\tilde{y} + Hll + Sq\overline{p} + a\tilde{y}\tilde{y} + B_0 a\overline{p}^2 \,. \tag{3.9}$$

Note the appearance of the additional (spurious) $\mathrm{SU}(2)$ global symmetry. This extra $\mathrm{SU}(2)$ is necessary to avoid the appearance of a Witten-anomaly in the Sp group. Note however, that none of the fields that transform under the $\mathrm{Sp}(2F-8)$ group transform under this $\mathrm{SU}(2)$. We should remark that there are many other possible ways to deconfine the anti-symmetric tensor both for N odd and N even, in which case the analog of the spurious global symmetry would be bigger, an $\mathrm{SU}(K)$ group. However, in the second dual one always ends up with an $\mathrm{Sp}(2F-8)$ group with the same matter content and none of these fields transforming under the $\mathrm{Sp}(2F-8)$ group will transform under the $\mathrm{SU}(K)$.

4. a-maximization in the deconfined dual theory: evidence for a mixed phase

In the final dual description, eq. (3.6), given that the mesons M and H are fundamental fields, their interactions are only dependent upon the couplings in the superpotential which contribute to their anomalous dimensions. The hypothesis in the electric description was that when M and H go free that they simply decouple while the rest of the theory remains interacting. Here we show evidence that this is in fact the case when M goes free, but not when the H meson goes free.

The deconfined dual description in eq. (3.6) is a strongly coupled description of the electric theory. In the electric picture there exists a weakly coupled Banks-Zaks fixed point [27], however in the deconfined dual description there is no weakly coupled fixed point due to the product group structure. Therefore one doesn't really know if the terms in the deconfined superpotential (3.7) are actually relevant or not. The best one can do is to assume the superpotential is relevant and thus deconfinement is valid around the Banks-Zaks fixed point of the electric theory. Then one can apply the a-maximization process to the final dual description to find the dimensions of the fields. By construction

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one then finds that the dimensions of the fields in the chiral ring of the "electric" theory agree with those of the final dual near the Banks-Zaks fixed point of the original theory. In this final dual description we find that M goes free at $x = x_M$ which corresponds identically to the electric description. When M goes free in the magnetic description the superpotential coupling in front of $Mql\tilde{y}$ must vanish. However, this does not imply any further field necessarily going free, and it is consistent to assume that due to this coupling flowing to zero M becomes a free field, while all other fields remain interacting. This is a very explicit realization of the procedure of [10] about incorporating accidental symmetries into a-maximization.

Thus it is reasonable to continue on with the assumption that when M goes free it simply decouples and we then find that H goes free at $x = x_H$, again exactly at the same point where it happens in the electric theory. However, the fact that H becomes a free field also implies that at this point its superpotential coupling flows to zero. Assuming continuity of the anomalous dimension in x we can also deduce that the dimension of the dual quark, $\Delta(l)$, is equal to one, since the superpotential term Hll has R-charge 2. The anomalous dimension of l can be expressed as $\gamma_l = g_{Sp}^2 h(\text{couplings})$ where h is a function of possibly all the couplings in the theory. The fact that the anomalous dimension of l vanishes at the point where H goes free could either be a consequence of an unexpected cancellation which causes there to be a zero of h when H goes free, or more simply it could imply that the gauge coupling of the Sp group also vanishes at this point. In the latter case l would also become a free field implying $\gamma_l = 0$ as required, but this also implies that the whole Sp group becomes free at the point where H goes free. This would be analogous to the case of SUSY QCD where the meson going free implied the whole dual gauge group became free, i.e. the theory entered a free magnetic phase. We find it more plausible that the Sp gauge group becomes free when H goes free. This is also the scenario that one would expect to happen based on estimates of the value of the β-function for the Sp group, which supports the claim that the Sp group gauge coupling in the IR indeed vanishes and Sp is IR free for sufficiently large x.

Next we will show how to actually estimate the value of the β-function for the Sp group. This will be similar to the argument found for the special case of five flavors in ref. [21]. We can simplify the analysis by noting that the ratio of the two holomorphic scales in the the final dual description Λ_{Sp} and Λ_{SU} can be varied arbitrarily. Because of holomorphy we know that there can be no phase transition as the ratio is varied, thus we can always go to a limit where one of the gauge couplings is as small as we desire. Thus we will work in the limit $\Lambda_{Sp} \ll \Lambda_{SU}$ and show that the β function of the Sp group is positive for large enough x, which will thus imply that the Sp group is indeed IR free. The bound on x that we find below is in accordance with the exact value found from a-maximization.

In the limit where g_{Sp} goes to zero we can expand the β function of the Sp group perturbatively in the small coupling limit:

$$\beta(g_{Sp}) = -\frac{g_{Sp}^3}{16\pi^2}\left[3(2F-6) - (F-3)(1-\gamma_{\tilde{y}}(g_{Sp}=0)) - (N+F-4)\right] + \mathcal{O}(g_{Sp}^5)\,. \quad (4.1)$$

Here we have used the fact that $\gamma_l(g_{Sp} = 0) = 0$. This follows from the fact that in the limit $g_{Sp} \to 0$ l is a gauge invariant, and it should obey $\Delta_l \geq 1$. However, it appears in a superpotential term Hll, where all three fields are gauge invariant in the zero Sp coupling limit. Since all of these fields have at least dimension one, this term has to be irrelevant, and so in this limit l is a free field, implying $\gamma_l(g_{Sp} = 0) = 0$.

We therefore see from (4.1) that if

$$4F - N - 11 + (F - 3)\gamma_{\tilde{y}} \leq 0 \tag{4.2}$$

the Sp group is IR free. The only thing we need to find is a bound on the anomalous dimension of $\tilde{y}$. We can get a bound on $\gamma_{\tilde{y}}$ in the following way: if we look at the last term in the superpotential (3.7) we see that the coefficient of this term by assumption does not vanish in the limit g_{Sp} goes to zero. Therefore the R-charge in this superpotential must add up to two. Assuming that the SU group is at a conformal fixed point we can use the relation between R-charges and anomalous dimensions to find

$$\gamma_a + 2\gamma_{\tilde{y}} = 0\,. \tag{4.3}$$

Thus we can get an upper bound on $\gamma_{\tilde{y}}$ if we find a lower bound on γ_a. Such a bound can be obtained by considering the unitarity bound for a gauge invariant containing only a's. Let us first suppose that F is odd, in this case $a^{\frac{F-3}{2}}$ is such a gauge invariant and maps to the field $B_F = Q^F A^{\frac{N-F}{2}}$ of the "electric" theory. The reason why we emphasize this is because the unitarity bound should only be imposed on fields that are part of the chiral ring. There do exist gauge invariant operators (for example $q\overline{p}$) that are lifted by the F-flatness conditions and therefore the unitarity bound should not be imposed on them. Since $B_F = Q^F A^{\frac{N-F}{2}}$ is a gauge invariant of the electric theory it is bounded by the unitarity constraint to have

$$\frac{F-3}{2} + \frac{F-3}{4}\gamma_a \geq 1\,, \tag{4.4}$$

since $\Delta_i = 1 + \gamma_i/2$. By combing eq. (4.3) and eq. (4.4) and taking the large F limit we see that

$$\gamma_{\tilde{y}} \leq 1\,. \tag{4.5}$$

If we use this bound for $\gamma_{\tilde{y}}$ and combine it with eq. (4.2) in the large N, F limit we see that for

$$N \geq 5F \tag{4.6}$$

or

$$x \geq 5 \tag{4.7}$$

the Sp gauge group is IR free. This is consistent with the result obtained from a-maximization, and gives strong support for the expectation that for $x > x_H$ the Sp group is indeed IR free. One should note that this bound is different than the naive estimate of where the Sp group would go free if you did not take into account the dynamics of the SU group. Ignoring the SU dynamics one would find that for $x \geq 4$ the Sp group was IR free but as we see including the effect of the superpotential and the SU strong dynamics we find a higher value of x at which Sp becomes free which is consistent with a-maximization.

For the case that F is even we will have to use a different gauge invariant to find a bound on $\gamma_{\tilde{y}}$. In this case we will use the invariant $qa^{\frac{F-4}{2}}$ which maps to $B_{F-1} = Q^{F-1}A^{\frac{N-F-1}{2}}$ of the electric theory which gives a unitarity constraint of

$$F - 4 + \gamma_q + \frac{F-4}{2}\gamma_a \geq 0\,. \tag{4.8}$$

Since eq. (4.3) does not depend on whether F is even nor odd, we can use eq. (4.3) with eq. (4.8) to show that

$$F - 4 + \gamma_q \geq (F-4)\gamma_{\tilde{y}}\,. \tag{4.9}$$

Using the a-maximization process we find that γ_q is bounded for all values of x therefore in the large F limit eq. (4.9) reduces to eq. (4.5). We therefore find that for F even or odd for $x > 5$ the Sp group becomes IR free. To summarize, we have found that there is a value of x above which the Sp gauge group is necessarily IR free. Since the anomalous dimension of l vanishes when H goes free, it is very plausible to identify the onset of the IR free phase of the Sp group with the point when H goes free, as argued when discussing the results of a-maximization. For the case when N is even one needs to use the dual in (3.8). We find identical results in this case in the large, N, F limit: the meson M goes free at x_M, H goes free at x_H and at that point the dual $\mathrm{Sp}(2F-8)$ group becomes IR free. One may worry about the appearance of a different SU gauge group and the additional SU(2) global symmetry in the final dual in this case. This is however not a real concern, since the fields that we claim to be free do not transform under the SU gauge symmetry nor under the extra global SU(2). One can also check, that our results about the free dual quarks and gauge fields are independent of the particular choice of the deconfining gauge group: the second dual will always have the same $\mathrm{Sp}(2F-8)$ factor with the same degrees of freedom transforming under it, which has to be the case if the Sp group is indeed IR free.

Thus what we have found is that there is a phase in the theory with the antisymmetric tensor which is similar to the free magnetic phase of SUSY QCD, since there is an IR free gauge group with free magnetic gauge bosons and free dual quarks. However, there are also many differences. The major difference is that in addition to the IR free gauge group there is another SU gauge group in the deconfined theory, which does not go free, and which in its matter content itself has an antisymmetric tensor just like the electric theory. If one assumes that as argued above the Sp group goes free, one can simply apply a-maximization to the remaining SU group to determine its evolution.

The results from a-maximization of the remaining SU group are shown in figure 3, where we plot the R-charges of the independent fields q, $\tilde{y}$ as a function of the number of flavors, F. Note, that the size of the original gauge group N no longer affects the dynamics of the SU group, which is now only a function of F, this is why we can present the results as a function of F only. However, it is still implicitly assumed that N is large enough for this mixed phase to occur at all because for $N < 8$ the H meson will not go free and thus there will not be a mixed phase. What we find is that none of the gauge invariants in the remaining SU group go free as we reduce the number of flavors before it confines at $F = 4$. What this suggests is that the mixed phase continues to exist down to $F = 5$ and once $F < 5$ the theory confines which signals the end of the mixed phase.

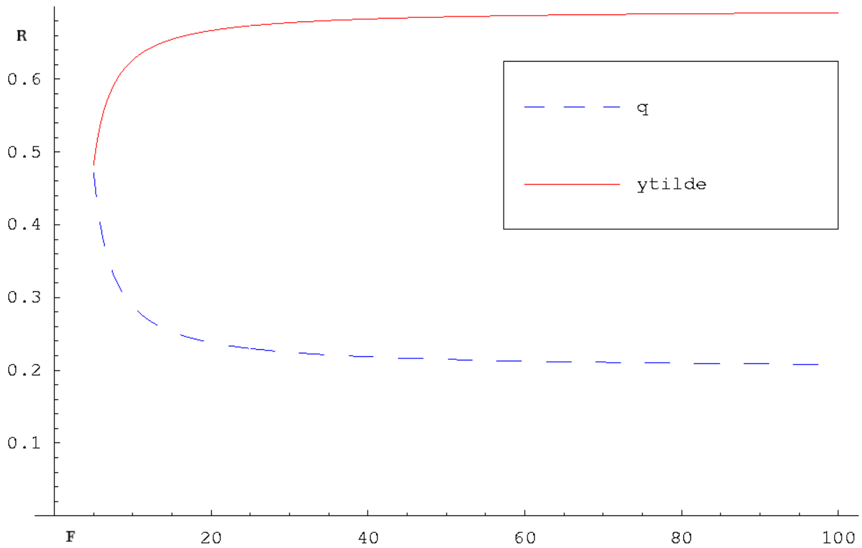

Figure 3: The R-charges of the independent fields are plotted as a function of F. The short dashed line represents the R-charge of q, and the long dashed line represents $\tilde{y}$. The smallest value that F can be is $F = 5$, for $F < 5$ this theory confines.

The mixed phase that occurs in this theory after H decouples is different than the normal picture that Seiberg duality shows for other theories. For instance in an SU(N) gauge theory with F flavors Seiberg duality shows that below $F = 3/2N$ the theory goes from a non-abelian Coulomb phase to a free magnetic phase. In the case of the antisymmetric tensor we have a product gauge group dual and only one of the gauge groups goes free.

5. Conclusion

The SU(N) supersymmetric gauge theory with a two-index antisymmetric tensor has many interesting properties not seen in other theories. There is no simple dual that unambiguously defines the phases of this gauge theory but combining "deconfinement" and the process of a-maximization we have determined the phase evolution of this theory in terms of N and F. This theory has a Banks-Zaks fixed point in the large N, F limit at $x = 1/2$. In the IR we believe that the theory is in an interacting non-abelian Coulomb phase near the Banks-Zaks fixed point. As one increases x the meson $M = Q\overline{Q}$ becomes a free field and decouples from the theory at $x = x_M \sim 2.95367$. Further increasing x will cause the other meson in the theory $H = \overline{Q}A\overline{Q}$ to become free at $x = x_H \sim 4.08952$. At the point where the meson H becomes free in the deconfined dual description it dictates that one of the dual gauge groups, Sp$(2F-8)$, becomes free. Therefore for $x > x_H$ the electric theory ceases to be a good description of the physics and one should use the dual deconfined description. In this deconfined description a mixed phase of the gauge theory continues to exist where the Sp$(2F-8)$ group remains free and the SU$(F-3)$ group is in an interacting non-abelian Coulomb phase. This mixed phase exists for all $F \geq 5$; at the point where $F < 5$ the theory confines and the dynamics is well understood. We expect similar results for SU(N) with a two-index symmetric tensor in the large-N, F limit, except the free gauge group will be SO($2F+8$). While a-maximization does not a priori tell us the phase structure of a theory, by combining it with the tools of deconfinement and duality we have shown that a new kind of fixed point can be reached.

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